FAR BEYOND

MAT122

Marginal Cost

Stony Brook University

Marginal Cost - Intro

$C(q)$ represents the cost of producing a quantity of q items.

Then $C'(q)$ would represent the **marginal cost**.

The cost to increase production from 'a' to 'b' units: $C(b) - C(a) = \int_a^b C'(q)\, dq$

The cost of producing 0 units: $C(0)$

Increase in cost between 0 units to 'b' units is called **total variable cost**. $\int_0^b C'(q)\, dq$

Total cost to produce 'b' units: $C(0) + \int_0^b C'(q)\, dq$

Marginal Cost - Example

ex. The marginal cost of drilling an oil well depends on the depth at which the drilling is done. Drilling becomes more expensive as it gets deeper into the earth.

currency in Saudi Arabia

The fixed costs total 1 million riyals and x is the depth, in meters.

Marginal costs are $C'(x) = 4000 + 10x$ riyals/meter.

Find the cost of drilling a 500m well.

$$C(0) + \int_0^b C'(q)\,dq$$

$$1{,}000{,}000 + \int_0^{500} (4000 + 10x)\,dx$$

$$= 1{,}000{,}000 + \left. 4000x + 5x^2 \right|_0^{500}$$

$$= 1{,}000{,}000 + (4000(500) + 5(500)^2 - 0)$$

$$= 1{,}000{,}000 + 2{,}000{,}000 + 5(250{,}000)$$

$$= 3{,}000{,}000 + 1{,}250{,}000 = \boxed{4{,}250{,}000 \text{ riyals}}$$

$$\int 10x\,dx = 10\frac{x^2}{2} = 5x^2$$

Differentials

recall:

$$\frac{dy}{dx} = f'(x)$$

differentials are dy and dx separately

can "solve" for dy

$$\frac{dy}{dx} = f'(x)$$

"multiply" to move dx to RHS

$$dy = f'(x)\,dx$$

ex. find the differential for $y = \left(1+x^3\right)^{-2}$

first take derivative: $\frac{dy}{dx} = -2\,(1+x^3)^{-3} \cdot 3x^2$

$$= -\frac{6x^2}{(1+x^3)^3}$$

next split dy/dx: $dy = -\frac{6x^2}{(1+x^3)^3}\,dx$

ex. find the differential for $y = e^{3t^2+1}$

take derivative: $\frac{dy}{dt} = 6t\,e^{3t^2+1}$

split dy/dt: $dy = 6t\,e^{3t^2+1}\,dt$